FAULT DIAGNOSIS BASED ON MULTIDIMENSIONAL FUZZY RELATIONAL EQUATIONS AND GENETIC ALGORITHM

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ABSTRACT

This paper proposes an approach for building fuzzy systems of diagnosis, which enables solving multidimensional fuzzy relational equations together with design and tuning of fuzzy relations on the basis of expert and experimental information.

1. INTRODUCTION

Fuzzy relational systems represent an alternative to rule based systems preserving their qualitative characteristics. Simulation of the cause-effect connections is done by way of interpreting Zadeh’s compositional rule of inference which connects input and output variables of an object (causes and effects) using a fuzzy relational matrix (Yager and Filev [1]). The fault diagnosis problem based on a cause and effect analysis is formally described by multidimensional fuzzy relational equations. The problem of inputs restoration and identification is formulated in the form of inverse fuzzy logical inference and requires solution of a system of fuzzy relational equations (Di Nola et al. [2]).

We propose an approach for building fuzzy systems of diagnosis, which enables solving fuzzy relational equations together with design and tuning of fuzzy relations on the basis of expert and experimental information. The essence of tuning consists of the selection of such membership functions of the fuzzy terms for the input and output variables (causes and effects) and such fuzzy relations, which provide minimal difference between theoretical and experimental results of diagnosis (Rotshtein [3]). The problem of solving multidimensional fuzzy relational equations amounts to the solution of an optimization problem using the genetic algorithm (Rotshtein and Rakytyanska [4]).
2. MULTIDIMENSIONAL FUZZY RELATIONAL EQUATIONS

We shall denote: \{x_1,\ldots,x_n\} is the set of input parameters; \{y_1,\ldots,y_m\} is the set of output parameters; \{c_{i1},\ldots,c_{ik_i}\} is the set of linguistic terms for parameter \(x_i\), \(i=\overline{1,n}\), evaluation; \{e_{j1},\ldots,e_{jq_{ij}}\} is the set of linguistic terms for parameter \(y_j\), \(j=\overline{1,m}\), evaluation.

Set \(\{C_1,\ldots,C_N\}=\{c_{11},\ldots,c_{1k_1}\},\ldots,\{c_{n1},\ldots,c_{nk_n}\}\) is called fuzzy causes (diagnoses), and set \(\{E_1,\ldots,E_M\}=\{e_{11},\ldots,e_{1q_1}\},\ldots,\{e_{m1},\ldots,e_{mq_m}\}\) is called fuzzy effects (symptoms).

Causes-effects interconnection is given by the system of SISO fuzzy relational matrices

\[ R_{ij} \equiv e_{ij} \times e_{jp} = [r_{il}, jp, i=\overline{1,n}, j=\overline{1,m}, l=\overline{1,k_i}, p=\overline{1,q_{ij}}] \]

which is equal to MIMO fuzzy relational matrix \(R \subseteq C_l \times E_J = [r_{IJ}, I=\overline{1,N}, J=\overline{1,M}]\).

An element of this matrix is a number \(r_{IJ} \in [0, 1]\), characterizing the degree to which cause \(C_I\) influences upon the rise of effect \(E_J\). Matrix \(R\) can be obtained on the basis of expert assessments (Rotshtein and Rakytyanska [4]).

In the presence of matrix \(R\) the «causes-effects» dependency can be described with the help of the extended compositional rule of inference (Yager and Filev [1])

\[
(\mu_{A_1}, \ldots, \mu_{A_n}) \ast 
\begin{bmatrix}
R_{11} & \ldots & R_{lm} \\
\ldots & \ldots & \ldots \\
R_{n1} & \ldots & R_{nm}
\end{bmatrix}
= (\mu_{B_1}, \ldots, \mu_{B_m}),
\]

(1)

where \((\mu_{A_1}, \ldots, \mu_{A_n})=((\mu_{c^{11}}, \ldots, \mu_{c^{1k_1}}), \ldots, (\mu_{c^{n1}}, \ldots, \mu_{c^{nk_n}}))\) or \(\mu^{C}=(\mu_{C_1}, \ldots, \mu_{C_N})\) is the fuzzy causes vector with elements \(\mu_{C_I} \in [0, 1]\), interpreted as some significance measures of \(C_I\) causes; \((\mu_{B_1}, \ldots, \mu_{B_m})=((\mu_{e^{11}}, \ldots, \mu_{e^{1q_1}}), \ldots, (\mu_{e^{m1}}, \ldots, \mu_{e^{mq_m}}))\) or \(\mu^{E}=(\mu_{E_1}, \ldots, \mu_{E_M})\) is the fuzzy effects vector with elements \(\mu_{E_I} \in [0, 1]\), interpreted as some significance measures of \(E_J\) effects; \(\ast\) is the operation of \((\circ, \bigwedge)\) (Yager and Filev [1]).

Finding vector \(\mu^{C}\) amounts to the solution of the system of multidimensional fuzzy relational equations, which is derived from relation (1):

\[
\mu_{B_j} = \bigwedge_{i=1,n}^{\circ}(\mu_{A_i} \circ R_{ij}), \quad j=\overline{1,m}.
\]

(2)

3. SOLVING MULTIDIMENSIONAL FUZZY RELATIONAL EQUATIONS

Following the approach, proposed in (Rotshtein and Rakytyanska [4]), the problem of solving fuzzy relational equations (2) is formulated as follows. Fuzzy causes vector \(\mu^{C}=(\mu_{C_1}, \ldots, \mu_{C_N})\) should be found which provides the least distance between observed and model measures of effects significances:
\[ F = \sum_{j=1}^{m} \left[ \mu^B_j - \bigcap_{i=1}^{n} (\mu^A_i \circ R_{ij}) \right]^2 = \min_{\mu^C} \]  

In the general case, the system (2) has several solution sets \( S_k(R, \mu^E) \), \( k = 1, K \), each of which is determined by the unique maximal solution \( \mu^C_k \) and the set of minimal solutions \( S_k^*(R, \mu^E) = \{ \mu^C_{kl}, l = 1, T_k \} \):

\[ S_k(R, \mu^E) = \bigcup_{\mu^C_{kl} \in S_k^*} \left[ \mu^C_k \bigcup \mu^C_{kl} \right], k = 1, K. \]  

Here \( \mu^C_k = (\mu_k^C_1, ..., \mu_k^C_N) \) and \( \mu^C_{kl} = (\mu_{kl}^C_1, ..., \mu_{kl}^C_N) \) are the vectors of the upper and lower bounds of causes \( C_I \) significance measures, where the union is taken over all \( \mu^C_{kl} \in S_k^*(R, \mu^E) \).

Formation of intervals (4) is accomplished by way of solving a multiple optimization problem (3) using the genetic algorithm (Rotshtein and Rakytyanska [4]).

4. FUZZY MODEL TUNING

It is assumed that the training data is given in the form of \( L \) pairs of experimental data:

\[ \left\{ \hat{X}_p, \hat{Y}_p \right\}, p = 1, L, \]  

where \( \hat{X}_p = (\hat{x}_{1p}, ..., \hat{x}_{np}) \) and \( \hat{Y}_p = (\hat{y}_{1p}, ..., \hat{y}_{mp}) \) are the vectors of the values of the input and output variables in the experiment number \( p \). In order to translate the specific values of the input and output variables into the measures of \( C_I \) causes and \( E_J \) effects significances we use a membership function of fuzzy terms in the form:

\[ \mu^T(u) = \frac{1}{1 + ((u - \beta) / \sigma)^2}, \]

where \( \beta \) is a coordinate of maximum, \( \mu^T(\beta) = 1 \); \( \sigma \) is a parameter of concentration.

The essence of tuning consists of finding such matrix \( R \) and such vectors of membership functions parameters \( B_C, \Omega_C, B_E, \Omega_E \), which provide the least distance between model and experimental fuzzy effects vectors:

\[ \sum_{p=1}^{L} [\mu^E(\hat{X}_p, R, B_C, \Omega_C) - \hat{\mu}^E(\hat{Y}_p, B_E, \Omega_E)]^2 = \min_{R, B_C, \Omega_C, B_E, \Omega_E} \]
5. EXAMPLE OF TECHNICAL DIAGNOSIS

Let us consider the algorithm’s performance having the recourse to the example of the fuel pump faults causes diagnosis. Input parameters are: \( x_1 \) – engine speed (30 – 50 r.p.m.); \( x_2 \) – inlet pressure (0.02 – 0.15 kg/cm\(^2\)); \( x_3 \) – drive gear clearance (0.1 – 0.3 mm); \( x_4 \) – fuel leakage (0.5 – 2.0 cm\(^3\)/h); \( x_5 \) – fuel kinematic viscosity ((55–170)·10\(^6\) m\(^2\)/c). Output parameters are: \( y_1 \) – productivity (17–22 m\(^3\)/h); \( y_2 \) – consumed power (2.1–3.0 kW).

Input term-assessments are: \( c_{11} \) – engine speed \( x_1 \) drop; \( c_{21} \) – inlet pressure \( x_2 \) drop; \( c_{31} \) – increase of clearance \( x_3 \), i.e. gear wear-out; \( c_{41} \) – increase of leakage \( x_4 \), i.e. fuel escape; \( c_{51} \) (\( c_{52} \)) – low (high) fuel viscosity \( x_5 \). Output term-assessments are: \( e_{11} \) (\( e_{12} \)) – productivity \( y_1 \) fall (rise); \( e_{21} \) (\( e_{22} \)) – consumed power \( y_2 \) drop (rise).

The «causes-effects» dependency is described with the help of the following fuzzy relational model

\[
(\mu^A_1,\mu^A_2) \ast \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} = (\mu^B_1,\mu^B_2),
\]

where \( \mu^A_1 = (\mu^{c_{11}},\mu^{c_{21}},\mu^{c_{31}},\mu^{c_{41}}) \), \( \mu^A_2 = (\mu^{c_{51}},\mu^{c_{52}}) \), \( \mu^B_1 = (\mu^{e_{11}},\mu^{e_{12}}) \), \( \mu^B_2 = (\mu^{e_{21}},\mu^{e_{22}}) \).

We shall define the set of causes and effects in the following way: \( \{ C_1, C_2, \ldots, C_6 \} = \{ c_{11}, c_{21}, c_{31}, c_{41}, c_{51}, c_{52} \} \); \( \{ E_1, E_2, \ldots, E_4 \} = \{ e_{11}, e_{12}, e_{21}, e_{22} \} \).

The system of multidimensional fuzzy relational equations is derived from relation (5):

\[
\begin{align*}
\mu^{E_1} &= [(\mu^{C_1} \land 0.41) \lor (\mu^{C_2} \land 0.65) \lor (\mu^{C_3} \land 0.80) \lor (\mu^{C_4} \land 0.62)] \land [(\mu^{C_5} \land 0.58) \lor (\mu^{C_6} \land 0.69)] \\
\mu^{E_2} &= [(\mu^{C_1} \land 0.59) \lor (\mu^{C_2} \land 0.09) \lor (\mu^{C_3} \land 0.23) \lor (\mu^{C_4} \land 0.45)] \land [(\mu^{C_5} \land 0.69) \lor (\mu^{C_6} \land 0.50)] \\
\mu^{E_3} &= [(\mu^{C_1} \land 0.92) \lor (\mu^{C_2} \land 0.88) \lor (\mu^{C_3} \land 0.50) \lor (\mu^{C_4} \land 0.79)] \land [(\mu^{C_5} \land 0.46) \lor (\mu^{C_6} \land 0.87)] \\
\mu^{E_4} &= [(\mu^{C_1} \land 0.09) \lor (\mu^{C_2} \land 0.09) \lor (\mu^{C_3} \land 0.68) \lor (\mu^{C_4} \land 0.11)] \land [(\mu^{C_5} \land 0.23) \lor (\mu^{C_6} \land 0.07)]
\end{align*}
\]

The observed parameters for a specific pump are: \( y_1^* = 19.3 \) m\(^3\)/h; \( y_2^* = 2.23 \) kW. The following measures of the effects significances correspond to these values:

\[
\begin{align*}
\mu^{E} &= (\mu^{E_1}(y_1^*) = 0.65; \mu^{E_2}(y_1^*) = 0.56; \mu^{E_3}(y_2^*) = 0.77; \mu^{E_4}(y_2^*) = 0.12.
\end{align*}
\]

Using the genetic algorithm (Rotshtein and Raktyanska [4]) we obtain the following results. The system of fuzzy relational equations has the four solution sets \( S_k (R, \mu^E) \), \( k = 1, 4 \), each of which can be represented in the form of intervals:
The resulting solution allows analyst to make the following conclusions. The cause of the observed pump state should be located and identified as the inlet pressure drop to $0.02 - 0.05$ kg/cm$^2$ with the fuel viscosity being high ($(140 – 170) \times 10^{-6}$ m$^2$/c), so that the significance measures of causes $C_2$ and $C_6$ are maximal. In addition, the observed state can be the effect of the engine speed drop to $30 – 40$ r.p.m. or the fuel leakage to $1.8 – 2.0$ cm$^3$/h with the fuel viscosity being high, since the significance measures of causes $C_1$ and $C_4$ are sufficiently high. The gear wear-out for the side clearance from $0.1$ to $0.15$ mm should be excluded, so that the significance measure of cause $C_3$ is small.

6. REFERENCES