Adaptive Azimuthal Null-Steering for a First-order Microphone Response

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***Abstract*—An azimuth steerable first-order superdirectional microphone response can be constructed by a combination of a monopole and two orthogonal dipole microphones. We derive a novel adaptive null-steering scheme based on the generalized sidelobe canceller (GSC), aiming to reject a single directional interference. To fully exploit the three microphone inputs, we use the extra degree of freedom to optimize the directivity index. Besides closed-form expressions for this optimal null-steering, we present a novel gradient-search strategy.**

I. Introduction

An azimuth steerable first-order superdirective beampattern can be constructed by combining an monopole microphone with two orthogonal dipole microphones. This class of small differential beamformers was first proposed in [1]. In [2], a beam-pattern synthesis method was presented for the rejection of a directional inference with the best possible diffuse noise reduction while having a unity response to a desired azimuthal direction φ_s. In this paper, we present an adaptive scheme based on the Generalized Sidelobe Canceller (GSC) that obtains this optimal beam-pattern synthesis automatically.

Via the monopole E_m and the two orthogonal dipoles E_d^x and E_d^y (having the main-lobe to respectively the x and y direction), we first rotate the dipoles via a rotation-matrix $\mathbf{R}_{\varphi_s}$:

$$\mathbf{R}_{\varphi_s} = \begin{bmatrix} \cos\varphi_s & -\sin\varphi_s \\ \sin\varphi_s & \cos\varphi_s \end{bmatrix}. \tag{1}$$

Next, two noise references E_{r_1} and E_{r_2} are constructed, being respectively a cardioid and a dipole having a zero response for angle φ_s (see Fig. 1) and the output response is given by:

$$E(\theta,\phi) = E_m - w_1 E_{r_1}(\theta,\phi) - w_2 E_{r_2}(\theta,\phi). \tag{2}$$

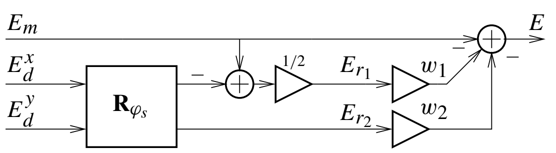

Fig. 1. GSC scheme using a monopole and two orthogonal dipoles.

The GSC weights are indicated with w_1 and w_2 and θ and ϕ are the standard spherical coordinates and:

$$\begin{aligned} E_m &= 1 && (3)\\ E_{r_1}(\theta,\phi) &= [1-\cos(\phi-\varphi_s)\sin\theta]/2 && (4)\\ E_{r_2}(\theta,\phi) &= \sin(\phi-\varphi_s)\sin\theta. && (5)\end{aligned}$$

Note that for any value of w_1 and w_2, a unity response at the output of the GSC is maintained for angle $\phi = \varphi_s$ and $\theta = \pi/2$. In Section II, we first analyze the non-adaptive GSC structure. In Section IV, we derive an adaptive algorithm based on the cost function of Section III. After the validation in Section V, conclusions are given in Section VI.

II. Optimal null-steering

Instead of computing the two GSC weights w_1 and w_2 by minimizing both the energy of the directional interference and other noises simultaneously, we hereafter minimize only the energy of the directional interference. As we have two GSC weights, we exploit the extra degree of freedom to control the beampattern in such a way that the directivity index (or directivity factor) is maximized.

Having a unity response for angle φ_s, we can compute the weights w_1 and w_2 to steer a zero toward any desired azimuthal angle φ_n, by solving $E(\theta,\phi) = 0$ for $\theta = \pi/2$. This results in the following relation between w_1 and w_2:

$$w_1 = \frac{2\,(1-w_2\sin\varphi)}{1-\cos\varphi} \Leftrightarrow w_2 = \frac{2-w_1\,(1-\cos\varphi)}{2\sin\varphi}, \tag{6}$$

with $\varphi = \varphi_n - \varphi_s$.

We exploit the extra degree of freedom to maximize the directivity factor Q_S, given by [4] [1] [1]:

$$Q_S = \frac{4\pi\, E^2(\pi/2,\varphi_s)}{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} E^2(\theta,\phi)\,\sin\theta\, d\theta\, d\phi}. \tag{7}$$

Using Eq. (6) in Eqs. (2)-(5) and computing the extrema of Eq. (7), we get the following solution for w_1 and w_2:

$$\begin{cases} w_1 = 2 - \frac{2}{3\cos\varphi+5} \\ w_2 = \frac{-\sin\varphi(3\cos\varphi+1)}{3\cos^2\varphi+2\cos\varphi-5}. \end{cases} \tag{8}$$

In Fig. 2, w_1 and w_2 are shown as function of φ.

The value of w_2 will be zero in case $\varphi = \pm 1.91$ (resulting in a hyper-cardioid) and in case $\varphi = \pi$ (resulting in a cardioid).

Via Eq. (8), we can obtain an expression for calculating φ_n:

$$\varphi_n = \varphi_s + s \arccos\Psi, \tag{9}$$

[1] All equations for spherical isotropic noise can be easily translated to the case for cylindrical isotropic noise.

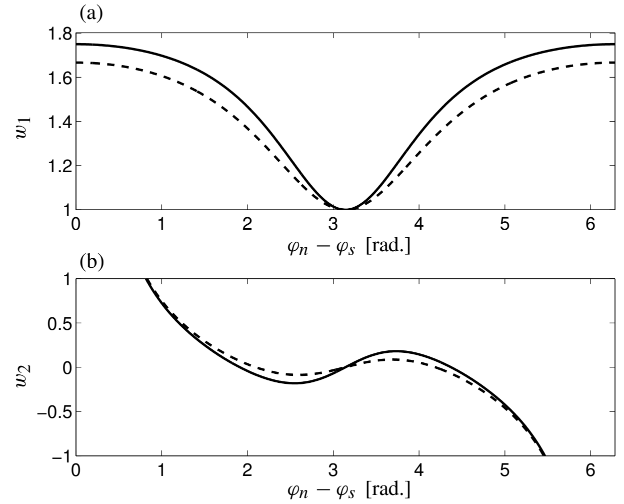

Fig. 2. Value of w_1 **(a)** and w_2 **(b)** as function of φ. The solid curves are the solutions for spherical isotropic noise, while the dashed curves are the solutions for cylindrical isotropic noise.

with the sign $s \in \{-1, 1\}$ and:

$$\Psi = \frac{5w_1 - 8}{-3w_1 + 6}. \tag{10}$$

The value of w_2 can be used to resolve the sign ambiguity in Eq. (9). By using Eq. (9) and using that $\sin(s \arccos X) = s\sqrt{1 - X^2}$, we can rewrite the rhs of Eq. (6) as:

$$w_2 = \frac{s\sqrt{1 - \Psi^2}\,(3\Psi + 1)}{3\Psi^2 + 2\Psi - 5}. \tag{11}$$

Knowing that $1 \le w_1 \le 1\frac{3}{4}$ and therefore $-1 \le \Psi \le 1$, we can derive that:

$$s = \begin{cases} +1 & \text{if:} \quad w_2(w_1 - 1\frac{1}{2}) \ge 0 \\ -1 & \text{otherwise,} \end{cases} \tag{12}$$

thereby solving the ambiguity.

III. Cost functions

A. Cost function for a directional interferer

We start with the discrete-time GSC equation[2], given by:

$$y[k] = m[k] - \hat{w}_1\, r_1[k] - \hat{w}_2\, r_2[k], \tag{13}$$

with the monopole signal $m[k] = s[k] + n[k]$, $s[k]$ the desired signal coming from angle φ_s, $n[k]$ the interferer coming from angle φ_n, $y[k]$ the output signal, and $r_1[k]$ and $r_2[k]$ the noise reference signals. For a single directional interferer and using the responses of Eq. (4)-(5), we obtain two reference signals given by:

$$r_1[k] = \left[(1/2) - (1/2)\cos\varphi\right] n[k], \tag{14}$$

$$r_2[k] = \sin\varphi\, n[k]. \tag{15}$$

[2] In this section, we use discrete-time signals indicated in lower-case, e.g. $x[k]$ with k the discrete-time index. Furthermore, estimated parameters are indicated with a hat, e.g. $\hat{w}[k]$, where sometimes the time-index k is omitted for convenience.

To reject a single interferer and at the same time have the best isotropic noise rejection, we know from the previous section that the weights $\hat{w}_1$ and $\hat{w}_2$ relate to $\hat{\varphi}_n - \varphi_s$ by:

$$\hat{w}_1 = 2 - \frac{2}{3\cos\hat{\varphi} + 5}, \tag{16}$$

$$\hat{w}_2 = \frac{-\sin\hat{\varphi}(3\cos\hat{\varphi} + 1)}{3\cos^2\hat{\varphi} + 2\cos\hat{\varphi} - 5}, \tag{17}$$

with:

$$\hat{\varphi} = \hat{\varphi}_n - \varphi_s, \tag{18}$$

where $\hat{\varphi}_n$ is the estimate of the angle of the undesired interferer.

The cost function $J(\hat{\varphi})$ is given by:

$$J(\hat{\varphi}) = \mathcal{E}\{y^2[k]\}, \tag{19}$$

with $\mathcal{E}\{\cdot\}$ the expectation operator.

Combining Eq. (13)-(19) and using $\mathcal{E}\{n[k]s[k]\} = 0$, we get after some computations:

$$J(\hat{\varphi}) = \sigma_s^2[k] + \left[\cos\varphi + \frac{\sin\varphi\sin\hat{\varphi}\left(3 + \frac{4}{\hat{A}}\right) - A}{3\hat{A} + 8}\right]^2 \sigma_n^2[k], \tag{20}$$

with $\hat{A} = \cos\hat{\varphi} - 1$, $A = \cos\varphi - 1$ and $\sigma_s^2[k] = \mathcal{E}\{s^2[k]\}$ and $\sigma_n^2[k] = \mathcal{E}\{n^2[k]\}$.

For $\sigma_n^2 = 1$ and $\sigma_s^2 = 0$, we plot the cost function for a few values of φ as shown in Fig. 3.

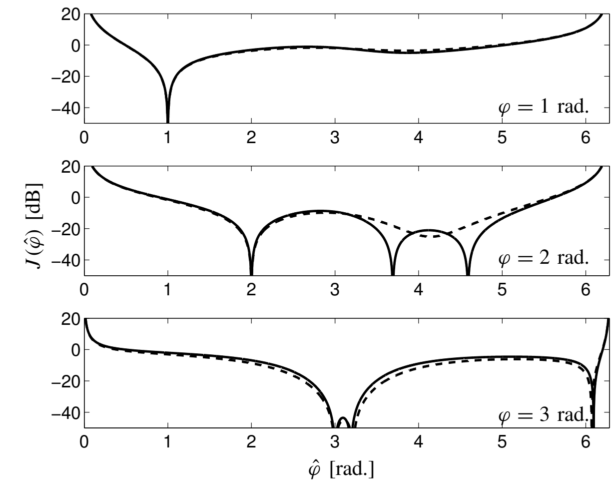

Fig. 3. Cost function $J(\hat{\varphi})$ (in dB) for a directional interferer: $\sigma_n^2 = 1$, for three values of φ. The solid curve is the cost function for spherical isotropic noise, by using $\hat{w}_1$ and $\hat{w}_2$ as in Eq. (16)-(17), while the dashed curve is the cost function for cylindrical isotrpic noise, where other expressions for $\hat{w}_1$ and $\hat{w}_2$ are used.

Note that for $\varphi = 0$ and $\varphi = 2\pi$, we get $J(\hat{\varphi}) = 1$, for which there are no minima and maxima.

B. Cost function for isotropic noise

It is also useful to analyze the cost function in the presence of isotropic noise. Assuming that we have a desired signal $s[k]$ coming from angle φ_s in the presence of isotropic noise and that there is no directional interferer, we obtain $m[k] = s[k] + u_1[k]$ and can compute the two noise reference-signals as:

$$r_1[k] = \left[u_1[k] - u_2[k]\sqrt{\gamma}\right]/2, \quad (21)$$
$$r_2[k] = u_3[k]\sqrt{\gamma}, \quad (22)$$

with $u_i[k]$, $i = 1, 2, 3$ mutually uncorrelated white-noise signals and with $\gamma = 1/3$ and $\gamma = 1/2$ to model respectively spherically and cylindrically isotropic noise [3] [5].

Using that $u_i[k]$, $i = 1, 2, 3$ are mutually uncorrelated and using the weights $\hat{w}_1$ and $\hat{w}_2$ as given by Eq. (16) and Eq. (17), we can write the cost function as:

$$J_d(\hat{\varphi}) = \sigma_s^2[k] + \left(\frac{\cos\hat{\varphi} - 15\gamma\cos\hat{\varphi} - 17\gamma - 1}{9\cos^3\hat{\varphi} + 21\cos^2\hat{\varphi} - 5\cos\hat{\varphi} - 25}\right)\sigma_d^2[k]. \quad (23)$$

The cost function ($\sigma_d^2 = 1$, $\sigma_s^2 = 0$) is shown in Fig. 4.

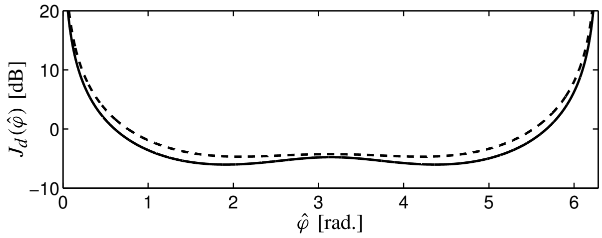

Fig. 4. Cost function $J_d(\hat{\varphi})$ (in dB) for isotropic noise: $\sigma_d^2 = 1$. The solid curve is the case with spherical isotropic noise $\gamma = 1/3$, while the dashed curve is the case with cylindrical isotropic noise $\gamma = 1/2$.

If we have both a directional interferer and isotropic noise and assume that $\mathcal{E}\{n[k]u_i[k]\} = 0$, we can construct the cost function based on superposition of the two cost functions.

IV. Gradient search algorithm

A. Computation of the gradient

A steepest descent update equation for $\hat{\varphi}$ can be derived by stepping in the direction opposite to the surface $J(\hat{\varphi})$ with respect to $\hat{\varphi}$:

$$\hat{\varphi}[k+1] = \hat{\varphi}[k] - \mu\nabla J(\hat{\varphi}), \quad (24)$$

with $\nabla J(\hat{\varphi})$ the gradient of the cost function $J(\hat{\varphi})$ w.r.t. $\hat{\varphi}$ and where μ is the update step-size with $0 < \mu < 1$. As in practice the mean $\mathcal{E}\{y^2[k]\}$ is not available, we have to compute an instantaneous estimate of the gradient $\hat{\nabla} J(\hat{\varphi})$. By using Eq. (13)-(18), we get:

$$\hat{\nabla} J(\hat{\varphi}) = \frac{\partial y^2[k]}{\partial\hat{\varphi}} = 2y[k]\begin{bmatrix} -\frac{\partial\hat{w}_1}{\partial\hat{\varphi}} & -\frac{\partial\hat{w}_2}{\partial\hat{\varphi}} \end{bmatrix}\begin{bmatrix} r_1[k] \\ r_2[k] \end{bmatrix}, \quad (25)$$

where the Jacobian elements are computed as:

$$-\frac{\partial\hat{w}_1}{\partial\hat{\varphi}} = D^{-1}\left[6\sin\hat{\varphi}\left(\cos\hat{\varphi} - 1\right)\right] \quad (26)$$
$$-\frac{\partial\hat{w}_2}{\partial\hat{\varphi}} = D^{-1}\left[3\cos^2\hat{\varphi} - 18\cos\hat{\varphi} - 17\right] \quad (27)$$

with:

$$D = 9\cos^3\hat{\varphi} + 21\cos^2\hat{\varphi} - 5\cos\hat{\varphi} - 25. \quad (28)$$

As the gradient-estimate clearly depends on the energy of $r_1[k]$ and $r_2[k]$, it is beneficial to normalize the update equation. While we could use power estimates for $r_1[k]$ and $r_2[k]$ for the normalization, in practice it is important that the adaptation is robust against desired signals $s[k]$. In case a desired signal is present, it is better to reduce the adaptation, as otherwise large misadjustments can occur in the adaptation. Hence, we use a normalized update-rule, given by:

$$\hat{\varphi}[k+1] = \hat{\varphi}[k] + \frac{2\mu y[k]}{\hat{P}_m[k] + \epsilon}\begin{bmatrix} \frac{\partial\hat{w}_1}{\partial\hat{\varphi}} & \frac{\partial\hat{w}_2}{\partial\hat{\varphi}} \end{bmatrix}\begin{bmatrix} r_1[k] \\ r_2[k] \end{bmatrix}, \quad (29)$$

where ϵ is a small value to prevent zero-division and the power-estimate $\hat{P}_m[k]$ of the monopole signal $m[k]$ is computed by a recursive averaging:

$$\hat{P}_m[k+1] = \beta\,\hat{P}_m[k] + (1-\beta)\,(m[k])^2, \quad (30)$$

where β is a smoothing parameter (lower, but close to 1).

B. Solutions for a directional interferer

When a directional interferer is present and we have no isotropic noise, it can be seen from Fig. 3 that the steepest descent method can end up in a non-unique minimum. Minimally two and maximally three minima can be found in this cost function. This can be clearly seen in Fig. 5, which illustrates the (local) minima of the cost function for different angles φ[3]. It can be seen that there is a minimum for the correct value ($\hat{\varphi} = \varphi$ for as indicated by the diagonal line). In addition it can be seen that at least one and possibly two other minima exist. For example, for $\varphi = 1$ rad., a cost function minimum exist at for the correct value $\hat{\varphi} = 1$ rad., but also at the wrong value of around 3.8 rad. Furthermore, for $\varphi \in [1.8; 4.2]$ rad., two wrong minima exist.

C. Solution for isotropic noise

When only isotropic noise is present and we have no directional interferer, we can compute from Eq. (23) that a minimum is obtained for $\hat{\varphi}$ given by:

$$\hat{\varphi} = \arccos\left(\frac{-6 - 54\gamma \pm 16\sqrt{6\gamma - 9\gamma^2}}{90\gamma - 6}\right). \quad (31)$$

Hence, for spherical isotropic noise, we obtain $\hat{\varphi} = \pi - \arccos(1/3) \approx \pm 1.91$. This resulting hyper-cardioid beam-pattern is known to be optimal in the presence of spherical isotropic noise conditions. When only isotropic noise is present, the optimal solution is automatically obtained via the gradient-search algorithm of Section IV-A. It is noted however, that the estimated value $\hat{\varphi}$ in the gradient-search algorithm for isotropic noise only situations, does not resemble the angle of a directional interferer anymore. Also when two (ore more) directional interferences are present, the angle will not be accurate anymore. We refer to [3] for improved solutions.

[3]It is possible to derive symbolic expressions for the 7 (local) minima/maxima of the cost function $J(\hat{\varphi})$ (also with complex-valued solutions). In Fig. 5 only the real-valued solutions are shown.

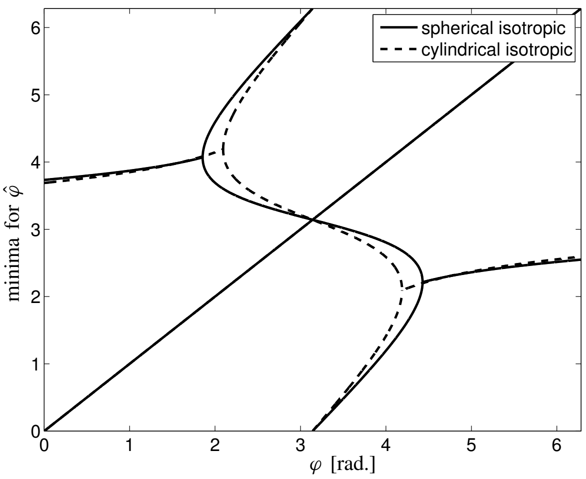

Fig. 5. Real-valued solutions (minima) for $\hat{\varphi}$ of $J(\hat{\varphi})$ as function of the directional source locations given by φ.

D. Proposed multiple gradient search strategy

From the previous section, we have seen that when we want to place a null to a directional interferer by means of an adaptive algorithm, multiple local optimal solutions exist (in case the diffuse noise is small or absent) and a standard gradient search strategy is not sufficient.

Looking at the situation when only a directional interferer is present, we can see from Fig. 5, that for an interferer angle in the interval $[0; \pi]$ (and similarly in the interval $[\pi; 2\pi]$), the minimum found by the gradient-search in the same interval is the correct value. This observation is exploited in a multiple gradient search strategy, where we have two estimates $\hat{\varphi}_1$ and $\hat{\varphi}_2$ obtained via separate gradient-searches running in two intervals $0 < \hat{\varphi}_1 < \pi$ and $\pi < \hat{\varphi}_2 < 2\pi$. In this way, we are sure that one of these estimates converges to the correct solution φ.

The selection of the correct estimate will be based on the gradient of the cost function evaluated at $\hat{\varphi} = \pi$ (corresponding to the weights $w_1 = 1$ and $w_2 = 0$), having a backward cardioid response with only a single null at the back. If this gradient is negative, the correct solution lies in interval $[0; \pi]$ and we choose $\hat{\varphi}_1$ as correct solution, while for a positive gradient, we choose $\hat{\varphi}_2$ as correct solution.

Via Eq. (25), an estimate of this gradient evaluated at $\hat{\varphi} = \pi$ can be computed as:

$$\hat{\nabla} J(\pi)[k] = -(m[k] - r_1[k])\, r_2[k]. \tag{32}$$

Hence, by correlating the forward cardioid response with the noise reference response E_{r_2} and looking at the sign, we can select the correct solution. As there can be noise present in the gradient $\hat{\nabla} J(\pi)$, we propose to smooth this estimate:

$$\hat{\nabla}_s J(\pi)[k+1] = \beta_s\, \hat{\nabla}_s J(\pi)[k] + (1-\beta_s)\, \frac{(r_1[k] - m[k])\, r_2[k]}{\hat{P}_m + \epsilon}, \tag{33}$$

with β_s a smoothing parameter (lower, but close to 1). To avoid a poor Directivity Index, we can also limit the range of $\hat{\varphi}$.

V. Validation

We validate the tracking behavior of the multiple gradient update algorithm as proposed in Section IV and perform a simulation, with a fixed desired source at position $\varphi_s = \pi/2$ rad. and variance $\sigma_n^2 = 1$. Furthermore, the angle of the directional interferer φ_n is linearly increased from 100 degrees to 440 degrees (i.e. a rotation of 340 degrees) in a time-span of 10000 samples. We also added isotropic noise ($\sigma_d^2 = 1/4$). Furthermore, we used $\alpha = 0.25$, $\mu = 0.02$, $\beta = 0.975$ and $\beta_s = 0.995$. The results are shown in Fig. 6. Note the resemblance of the curves in Fig. 6 with the theoretic minimum solutions of the cost function in Fig. 5.

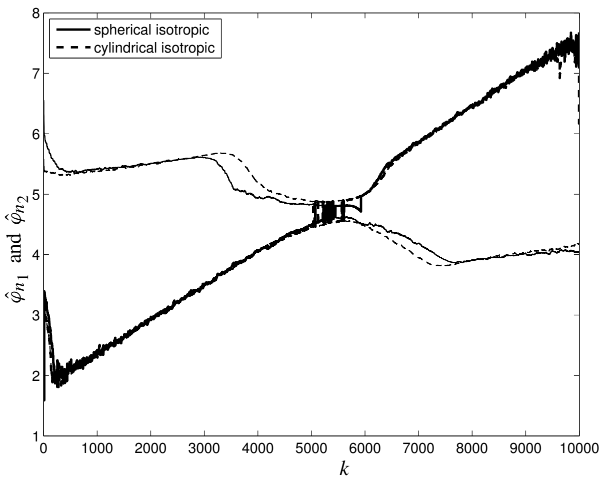

Fig. 6. Simulation with a directional interferer, where $\hat{\varphi}_{n_1} = \hat{\varphi}_1 + \varphi_s$ and $\hat{\varphi}_{n_2} = \hat{\varphi}_2 + \varphi_s$, $\sigma_n^2 = 1$ and $\sigma_d^2 = \frac{1}{4}$ (spherical isotropic noise); the bold-curve indicates the correct solution selected via the gradient estimate $\hat{\nabla}_s J(\pi)$.

VI. Conclusions

An adaptive first-order superdirectional beamformer was presented, having a unity response for a desired azimuthal angle, while placing a null at an undesired azimuthal angle under the constraint that the directivity index is maximized. The algorithm is based on a cost function that is minimized via a gradient search algorithm. As the cost function has multiple (local) minima, we propose a multiple gradient search algorithm that tracks two candidate-angles for the directional interferer. Via a third gradient estimate, the correct angle from these two candidate-angles is selected for the null-steering.

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